

THE PUZZLE CORNER

BY

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Readers are invited to send comments, and to send exercises, even if they don't know the answer. Write to Laurent.Rosaz@lri.fr.

86 Try to catch the largest

Let N be an integer. You know what number N is.

I will pick up a permutation σ of $[1..N]$ at uniform random (each of them may come out with probability $1/n!$). Then I will choose N numbers $x_1 < x_2 < \dots < x_N$. During the next N minutes, we will do the following: At the beginning of the i^{th} minute, I will tell you what $x_{\sigma(i)}$ is. As long as the minute hasn't ended, you are allowed to claim that the number I just gave to you is the largest one (i.e. x_N). You are allowed only one guess in the game. If you make a guess and if it is right, you win, otherwise, you lose. Because I choose the numbers, all what you can rely on are the comparisons between numbers. What is your best strategy ?

Example. If $N = 3$, you may claim that the first number is the largest, which makes you win with probability $1/3$. Or you can do the following: Make no claim in the first minute. If the second number is larger than the first one, then claim the second one is the largest, else if the third one is larger than the first two, then claim the third one is the largest, else you have lost anyway. This latter strategy is better since it makes you win with probability $1/2$ (if the permutation is among $\{(1, 3, 2), (2, 3, 1), (2, 1, 3)\}$).

SOLUTIONS TO PREVIOUS PUZZLES

85 What's next ?

A simple rule leads to the following sequence :

$u_0 = 4, u_1 = 3, u_2 = 3, u_3 = 5, u_4 = 4, u_5 = 4, u_6 = 3, u_7 = 5, u_8 = 5, u_9 = 4,$
 $\dots, u_{100} = 10, \dots, u_{999} = 24, u_{1000} = 11, \dots$

What is u_{10} ?

Solution u_n is the number of letters used to write number n in english. So $u_{10} = 3$.