
Abstract of PhD Thesis

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Abstract

In this thesis we propose a new approach to deduction methods for temporal logic. Our proposal is based on a non-customary inductive definition for eventualities, that allows us to provide dual systems of tableaux and sequents for Propositional Linear-time Temporal Logic (PLTL). Then, we extend our approach to the resolution framework and we present a clausal temporal resolution method for PLTL. Finally, we make use of this new resolution method for establishing logical foundations for declarative temporal logic programming languages.

The key element in the deduction systems for temporal logic is to deal with eventualities and “hidden” invariants that may prevent the fulfillment of eventualities. Traditional tableau systems for PLTL generate an auxiliary graph in a first pass. Then, in a second pass, the fulfillment of eventualities is checked and unsatisfiable nodes are pruned. The one-pass tableau introduced by S. Schwendimann requires an additional handling of information in order to detect cyclic branches that contain unfulfilled eventualities. In traditional sequent calculi for PLTL, the issue of eventualities and hidden invariants is mainly tackled by using invariant-based rules or infinitary rules. A remarkable consequence of using the above mentioned approaches in the tableau and sequent frameworks, is that temporal logic fails to carry out the classical correspondence between tableaux and sequents. In this thesis, we first provide the one-pass tableau method τ_{TM} that instead of a graph obtains a cyclic tree to decide whether a set of formulas is satisfiable. In τ_{TM} tableaux are classical-like. For unsatisfiable sets of formulas, τ_{TM} produces tableaux whose leaves contain a formula and its negation. In the case of satisfiable sets of formulas, τ_{TM} builds tableaux where each fully expanded open branch characterizes a collection of models for the set of formulas in the root. The tableau method τ_{TM} is complete and yields a decision procedure for PLTL. This tableau method is directly associated to the one-sided sequent calculus τ_{TC} and also to the two-sided sequent calculus g_{TC} . Likewise τ_{TM} , the two calculi τ_{TC} and g_{TC}

are sound and complete and also are free from all the structural rules that hinder the mechanization of deduction, e.g. weakening, contraction and cut (including invariant-based cut). Therefore, we show that the classical correspondence between tableaux and sequent calculi can be extended to temporal logic.

The most fruitful approach in the literature on resolution methods for temporal logic, due to M. Fisher, deals with PLTL and requires to generate invariants for performing resolution on eventualities. In this thesis, we present a new approach to resolution for PLTL. The main novelty of our approach is that we do not generate invariants for performing resolution on eventualities. Our temporal resolution method TRS is based on the methods of tableaux and sequents mentioned above and involves an effective translation of any formula into its clausal normal form. TRS is sound and complete. This method is also terminating, hence it gives rise to a new decision procedure for PLTL.

Finally, we present the declarative propositional temporal logic programming language TeDiLog that is a combination of the temporal and disjunctive paradigms in Logic Programming. We formally define operational and logical semantics for TeDiLog and prove their equivalence. The operational semantics of TeDiLog is based on TRS. TeDiLog is very expressive, in particular, it allows both eventualities and always-formulas to occur in clause heads and also in clause bodies.

Since the tableau method presented in this thesis is able to detect that the fulfillment of an eventuality is prevented by a hidden invariant without checking for it by means of an extra process, since our finitary sequent calculi do not include invariant-based rules and since our resolution method dispenses with invariant generation, we say that our deduction methods are invariant-free.

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