Abstract of PhD Thesis

Author: Sergi Oliva
Title: On the Complexity of Resolution-based Proof Systems
Language: English
Supervisor: Albert Atserias
Institute: Universitat Politècnica de Catalunya – BarcelonaTech
Date: 2 May 2013

Abstract

Propositional Proof Complexity is the area of Computational Complexity that studies the length of proofs in propositional logic. One of its main questions is to determine which particular propositional formulas have short proofs in a given propositional proof system. In this thesis we present several results that answer a particular case of this question or are intimately related to it, all on proof systems that are extensions of the well-known resolution proof system.

The first result of this thesis is that TQBF, the problem of determining if a fully-quantified propositional CNF-formula is true, is PSPACE-complete even when restricted to instances of bounded tree-width, i.e. a parameter of structures that measures their similarity to a tree. Instances of bounded tree-width of many NP-complete problems are tractable, e.g. SAT, the boolean satisfiability problem. We show that this does not scale up to TQBF. We also consider Q-resolution, a quantifier-aware version of resolution. On the negative side, our first result implies that, unless NP = PSPACE, the class of fully-quantified CNF-formulas of bounded tree-width does not have short proofs in any proof system (and in particular in Q-resolution). On the positive side, we show that instances with bounded respectful tree-width, a more restrictive condition, do have short proofs in Q-resolution. We also give a natural family of formulas with this property that have real-world applications.

The second result proposes a semantic way to compare first-order principles with respect to the length of the proofs of their propositional translations in a particular propositional proof system. To do that, we use the classical logic concept of interpretability. Informally, we say that a first-order formula can be interpreted in another if the first one can be expressed using the vocabulary of the second, plus some extra features. We show that first-order formulas whose propositional translations have short R(const)-proofs, i.e. a generalized version of resolution with DNF-formulas of constant-size terms, are closed under a weaker form of in-

terpretability (that with no extra features), called definability. Our main result is a similar result on interpretability. Also, we show some examples of interpretations and show a systematic technique to transform some Σ_1-definitions into quantifier-free interpretations.

The third and final result is about a relativized weak pigeonhole principle. This says that if at least $2n$ out of n^2 pigeons decide to fly into n holes, then some hole must be doubly occupied. We prove that the CNF encoding of this principle does not have polynomial-size DNF-refutations, i.e. refutations in the generalized version of resolution with unbounded DNF-formulas. For this proof we discuss the existence of unbalanced low-degree bipartite expanders satisfying a certain robustness condition.

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Author's correspondence address Sergi Oliva
Edifici Ω – Office S108
Jordi Girona Salgado, 1-3
08034 – Barcelona
Spain
Email: *oliva@lsi.upc.edu*