THE THEORY BLOGS COLUMN

BY

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Bill Gasarch is a professor of computer science at the University of Maryland at College Park. He works on computational complexity and combinatorics, and he is interested in math education. Bill writes with Lance Fortnow (who was featured in an earlier column) the “Computational Complexity Blog.”

In his guest column, Bill answers our questions on his experience writing for a theory blog with a very large and engaged community of readers, he tells us about his sources of inspiration, and he highlights two posts from his archives: one on open problems in mathematics and another on using SAT solvers to get concrete bounds on extremal combinatorics problems.

Computational Complexity

A Conversation with Bill Gasarch

(If you are reading this in pdf then you can click on the links in this article by clicking on the "*here*" of "*see here*".)

Q: Bill, thanks for taking the time for this conversation. Can you tell us how your collaboration with Lance Fortnow on the Computational Complexity Blog got started?

Lance started complexityblog on August 22, 2002. Lance invited me to do a Complexitycast with him for Dec 12, 2006 (see here). He went on vacation in January of 2006 and asked me to guest post for the week. My first guest post was titled *Are you a Luddite* (see here and follow up posts see here and see here).

I began doing a few more guest posts. On March 25, 2007 Lance suddenly decided that he said all he wanted to say, so he retired (temporarily as it turned out) from blogging (see here).

He got several blog posts and emails saying that the blog SHOULD go on. I was the only person in the intersection of WANT TO DO IT and COULD DO IT. On March 30, 2007 I blogged *I am the new Complexity Blogger* (see here). My first real post was on *What to make of the Ind of CH?* since Paul Cohen had died recently (see here). I revisited this topic in 2020 (see here).

On January 18, 2008 Lance decided he had more to say and announced that he was coming back to the blog (see here though its at the end of that post). We have been co-blogging every since.

Q: Your writing style is very idiosyncratic, and when I read a post in the Computational Complexity Blog I can always tell if it was written by you or by Lance. Do you have any inspirations or models for your writing?

I list some of my influencers.

1. My hobby is comedy and novelty songs (I have a large collection) so I have absorbed some from that realm. Hence I keep it light (with pointers to more serious material), try to use clever wordplay, and know when to stop (its a maxim among comedians that you should tell the same joke twice, but no more than that).

2. Lance obviously influences me. He started the blog and set the tone for it.

3. The following authors all influenced me

   (a) In High School I read many books by Martin Gardner on recreational math (I find the difference between recreational math and serious math to be either thin or non-existent). I liked his style and content.

   (b) As an adult I read Brian Hayes's *Group Theory in the Bedroom* which is not as sexy as it sounds, but does explain math to the layperson.

   (c) Ian Stewart's book *Ian Stewart's Cabinet of Mathematical Curiosities* stands out for style since some chapters are long, some are short, some are funny, some are serious, some are doing math, some are commenting on math and some are not-quite-math. This influences me to NOT feel the need to be uniform.

       As an example of an entry that was short, funny, and not-quite-math, here is a poem from the book:

       A challenge for many long ages
       Had baffled the savants and sages
           Yet at last came the light
           Seems that Fermat was right
       To the margin add 200 pages

       While trying, without success, to find the origin of that poem, I found many great poems about Fermat's last theorem, see here.

       ADDED LATER: In my attempt to find the origin of the poem I looked at Ian Stewart's book. It was not there! I then looked at another Ian Stewart book. It was not there either! However, the poem is similar to chapters that are in the book, so I keep the poem in this column.

       ADDED LATER: When a commenter corrects an error on a blog post I want to make the correction but NOT hide that I made a mistake. So I add *ADDED LATER* and explain the situation.

   (d) I read Doug Hofstadter's *Godel-Escher-Bach* in the summer between undergrad school and graduate school. I knew JUST ENOUGH logic to understand it but NOT SO MUCH as to be bored. I try to hit that sweet spot in my blogs as well.

   (e) I wrote joint book review of books by Gardner, Hayes, and Stewart. See here. I wrote a book review of *Martin Gardner in the Twenty First Century* which is about serious math he inspired. See here. I should reread *Godel Escher Bach* and write a review of it.

4. My Darling knows enough computer science and math to know what I am talking about (she has a Masters Degree in Computer Science) but not enough to have drunk the Kool-aid. This has influenced some posts. The most obvious is that when I told her the Banach-Tarski Paradox she declared *Math is broken!* She may be right. The BT paradox was the subject of a blog post (see here) and has been mentioned in other posts.

Q: You often solicit reader's opinions or feedback on your posts, and you engage with them. Can you recall some memorable exchange that took place following one of your posts? Has your research ever been influenced by such discussions?

The comments I get inspire me to READ up on some topics, which helps my research indirectly. More likely what I get out of the comments is

1. material for my open problems column (I am currently the SIGACT News Open Problems Column Editor),
2. surveys,
3. books to read and write review of,
4. problems for the Maryland High School Math Competition,
5. blog posts which will give me
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      v. blog posts which will give me ...

Even though I was trained as a mathematician I will go against that training and give some *examples*. I give summaries of blog posts in italics with a pointer to the original post, and then some comments on the comments.

(See here) Alice, Bob, Carol each have an n-bit number on their foreheads and they want to know if the sum is $2^n - 1$. *They can do this with n bits of communication. Can they do the problem with less bits of communication? This sounds like a FUN problem to tell your undergraduates about. Chandra-Furst-Lipton [1] showed that, for large n, they can do it in* $\sqrt{n}$ *bits. The proof uses large 3-free sets (from Ramsey Theory) which I find fun, but your typical undergraduate might not. Is there a way to make this problem FUN by finding a way to do it with (say)* $\frac{n}{10}$ *bits but in a way undergraduates can understand?*

Dean Foster left a comment on that blog post which gave an an elementary $\frac{n}{2}+O(1)$ solution. Now that I have a starting point I will write an open problems column where I ask how well we can do using elementary methods. Lower bounds are impossible here since *elementary methods* is not rigorous; however, upper bounds would be great. I also (easily) extended the solution to k people and $\frac{n}{k-1} + O(1)$.

2) *(See here) Hilbert's 10th problem is to (in todays terms) find an algorithm that will, given a poly* $p(x_1, \ldots, x_n) \in \mathsf{Z}[x]$, *determine if there is a solution in* Z. *From the work of Davis, Putnam, Robinson, and Matiyasevich the problem is known to be undecidable. Let* $H10(d, n)$ *be the problem where the polynomial is of degree d and has n variables. There should be a grid of* (d, n) *saying, for each entry, if its undecidable (U), decidable (D), or unknown (UK). But there is not. Darn.*

The comments on this blog inspired me to do a survey of what is known, which appeared in the BEATCS algorithms column [2]. A later version is on arxiv (see here). An email from a reader was very enlightening and I quoted it in the arxiv version:

Timothy Chow offered this speculation in an email to me: One reason there isn't already a website of the type you envision is that from a number-theoretic (or decidability) point of view, parameterization by degree and number of variables is not as natural as it might seem at first glance. The most fruitful lines of research have been geometric, and so geometric concepts such as smoothness, dimension, and genus are more natural than, say, degree. A nice survey by a number theorist is the book Rational Points on Varieties by Bjorn Poonen [4] Much of it is highly technical; however, reading the preface is very enlightening. Roughly speaking, the current state of the art is that there is really only one known way to prove that a system of Diophantine equations has no rational solution.

Timothy's email lead to a blog post, where I quote him, about who should decide what problems are natural to work on (see here).

Q: I know you are quite interested in mathematics education and about getting K-12 kids interested in mathematical thinking and research. What role do you think that blogs can play in getting young people interested in mathematics?

This has two answers.
1) If a K-12 student reads my blog (probably High School, though I do know one 9 year old who reads it) then since it's light and we highlight the ideas it may inspire them to read something more serious, or to contact Lance or I (some have contacted me).
2) Because I write the blog, I have become good at coming up with ideas for High School projects. Because I am good at coming up with ideas for High School projects, I write the blog. The cliche comment to ask *which came first, the chicken or the egg?*; however, this cliche is no longer accurate since *the chicken came first* (see here). In any case, no matter how you look at it, I have lots of ideas for High School Projects and for blog posts.

Q: Can you highlight one post from the past and tell us about it?

I will highlight a few posts.

1) My advisor Harry Lewis emailed me that his 9 year old granddaughter Alexandra wants to know what happens when you get a Millennium prize so that she will be ready.
This inspired some thoughts:

1. Is Alexandra ready for P vs NP?
2. Is P vs NP ready for Alexandra?
3. How likely is it that Alexandra will resolve P vs NP (or if she is rebellious, the Navier-Stokes equation)?
4. How much progress as a community have we made on P vs NP? Not much. That topic has been blogged about and discussed a lot before, so no need to rehash that topic.
5. Erdős has said of the Collatz Conjecture that

   Mathematics is not yet ready for such problems

   Alexandra and Erdős jointly inspire the following question:

   For which math problems is it the case that, when they were posed, Mathematics was not yet ready for such problems?

The post (see here) was a historical tour-de-force of open or previously open problems in mathematics, examining this question.

The most intriguing was the three problems of antiquity: can one, with just a ruler and a compass (as a kid I wondered why knowing what direction north was would help with geometry) trisect an angle, double a cube, or square the circle? When it was posed

Mathematics was REALLY not ready for these problems!!

Other problems I looked at were Fermat's Last Theorem, Completeness of Peano Arithmetic, the Continuum Hypothesis, Hilbert's 10th problem, The Four Color problem, Poincare's Conjecture, The Erdős Distance problem, The Collatz Conjecture (spoiler alert: Math is still not ready for it), Ramsey of 5 (Math is still not ready for it but there is a bigger issue: no interesting mathematics has been found in pursuit of finding Ramsey of 5), and the Twin Primes Conjecture.

The post ended with the note that Alexandra was going to work on Collatz over the summer, and I wished her luck.

The post inspired me to read the book *Tales of the Impossible: The 2000 year quest to solve the mathematical problems of antiquity* by David Richeson, which was very enlightening. For my review of it in SIGACT News see here.

2) If n is a natural number then $[n]$ is $\{1,\ldots,n\}$. An $n \times m$ grid is *c-colorable* if there is a map from $[n] \times [m] \to [c]$ so that there are no rectangles where all four corners are the same color. I was working on the following problem (with co-authors Stephen Fenner, Charles Glover, Semmy Purewal): for which n, m, c is $n \times m$ c-colorable? We had determined exactly which grids were 2-colorable. We had determined exactly which grids were 3-colorable. We had reason to think that 17×17 IS 4-colorable. But we could not prove it. On November 30, 2009 I posted the following (I am paraphrasing):

I offer a bounty of \$289.00 *to the first person to email me a 4-coloring of the* 17×17 *grid. (See here.)*

Brian Hayes saw this and popularized the challenge in his column. His column has far more readers than mine does, so this got the problem more out there. Several people told me *just throw a SAT SOLVER at it*. Those who tried had no success. But finally . . .

In 2012 Steinbach and Posthoff [5, 6, 7] obtained the coloring (and a few others that I needed) and I happily paid them the \$289.00. I blogged about it (see here). I was thus able to solve exactly which grids were 4-colorable. The problem of 5-colorability seems to be beyond todays technology and might always be; however, with current progress in AI, I could be surprised.

This is the most direct case of me posing a problem on the blog and getting it answered. For the final paper see here.

3) On April 1, 2010 I posted a manifesto that the theory community should STOP proving that our techniques won't suffice to solve certain problems (e.g., oracles, natural proofs) and PROVE SOMETHING. I laid down some problems where progress seemed possible.

1. Prove that NP is different from time $2^{O(n)}$.
2. Determine how NL and P compare.
3. Determine how deterministic primitive recursive and nondeterministic primitive recursive compare.
4. Determine how deterministic finite automata and nondeterministic finite automata compare.

I stated that these problems should be solvable with the methods available to us since neither oracles nor natural proofs rule out current techniques. So progress is possible.

Some astute readers of this column may notice, as some astute readers of the original post noticed, the following:

a) All four open problems have been solved a long time ago. Problems 2 and 4 are well known results. Problems 1 and 3 would make good homework problems for an undergraduate course that covers this material.

b) The post was made on April 1 which is also known as *April Fools Day*. While writing this column I looked up *April Fools Day* and found out that its celebrated across the world and not just in America. (see here). I did not know that. So even writing a column *about* the blog enlightened me!

I am particularly proud of this post since it has become a cliche to post on April fools day that some *open* problems are *solved*. I turned that around: I posted that some *solved* problems are *open*.

Q: Is there something else that you would like to share with our readers?

Some random thoughts:

1. Early on Lance told me to NOT worry about comments. Engage YES, but do not write a post thinking *this will get a lot of comments!* Unlike todays journalists (or perhaps social media aggregator) we are not paid by-the-click. We write what we find interesting and hope others will, but are not obsessed with that part. To quote Ricky Nelson's song Garden Party *you can't please everyone, so you've got to please yourself.* For the video on you tube of that song, see here. That video gets 10,000 likes which Ricky Nelson, if he was alive and consistent, would not care about.

2. Blogs are a wonderful place to exchange ideas without referees or program committees getting in the way.

3. Clyde Kruskal and I have written a book, *Problems with a point: Exploring Math and Computer Science* (see here), based on some of my blog posts. We took posts that made a point about Math or CS, and then did some Math or CS to illustrate the point. The essays in the book are polished and completed versions of the posts, or in some cases a set of posts.

4. While the blog is called *complexityblog* we do not feel constrained by that. There are posts on CS, Math, academia, anything really. When Jimmy Buffett passed away I had a blog (see here) about songs that have a *contradiction* (thats relevant for logic!) between what the *lyrics say* and what people *think they say* since Jimmy Buffett's *Margaritiville* is a great example.

5. This column has 29 links in it (one is in the bibliography). Is that a BEATCS record? The blog post with the most links was titled *Disproving that Myth that many early logicians were a few axioms short of a complete set* (see here) which had 71 links.

References

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[2] W. Gasarch. Hilbert's tenth problem for fixed d and n. *Bulletin of the European association of theoretical computer science (BEATCS)*, 133, 2021. See the arxiv article LINK.

[3] W. Gasarch and C. Kruskal. *Problems with a point: exploring math and computer science*. World Scientific, 2019.

[4] B. Poonen. *Rational points on varieties*, volume 186 of *Graduate studies in mathematics*. American Mathematical Society, 2017.

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[7] B. Steinbach and C. Posthoff. Utilization of permutation classes for solving extremely complex 4-colorable rectangle-free grids. In *Proceedings of the IEEE 2012 international conference on systems and informatics*, 2012.