Obituary
Gregory Tseytin

On August 27, 2022, Gregory Tseytin, a brilliant Russian and American logician and computer scientist, passed away[1]. Tseytin was born in Leningrad, USSR (now St. Petersburg, Russia) on November 15, 1936. His mother was an engineer economist and his father a teacher of mathematics. Both parents came from large Jewish families in Belarus. During the war, the mother and son were evacuated to the Kirov (now Vyatka) region. Gregory was a child prodigy. In 1945, when Gregory entered the fifth grade, a committee, comprising renowned mathematicians G. M. Fichtenholz, V. A. Tartakovsky, and D. K. Faddeev, concluded that "Gregory's development in mathematics is extraordinary. He is fluent in the material taught in high school ..." At age 9, Gregory won the first prizes in the high school competitions in mathematics and physics. At school, he showed great interest in languages. He was also excellent at skating. In 1951, he graduated from high school with the highest honors but, because of his age, he was not admitted to Leningrad State University (LSU). So he just audited the lectures. In September 1952, with the permission of the Ministry of Education and supported by the LSU rector A.D. Alexandrov, he was admitted as a second-year student.

Those around Gregory were pleasantly surprised by his youthful innocence and his thirst for contacts. But it was difficult for him to socialize with his classmates. He was younger than his fellow students but knew much more. His mother accompanied him to the university and waited for him there until the classes were over. Also, he adhered strictly to formal rules in speech and actions. Eventually he adjusted to student life and became very active in the student scientific society, published a student mathematical journal, took part in organizing an English club, released the first rotaprint collection of student and tourist songs, and worked at student construction sites. In the meantime, he also graduated from an adult music school. Later, in 1960, he was among the organizers and teachers of the Youth Mathematical School.

[1] A version of this obituary was published in the Russian journal Успехи Математических Наук 78:3 (2023) 170–176. (The journal used to be translated until recently as *Russian Mathematical Surveys*).

In his second university year, Gregory attended Andrei A. Markov Jr.'s lectures on the theory of algorithms and realized that he found his teacher. Mathematical logic suited his way of thinking like no other area. His first mathematical work "On the number of steps in an algorithm" was completed by the end of the second year. Probably, this student result became part of the paper [5]. Tseytin's diploma thesis "Associative calculi with undecidable equivalence problems" was published in Reports of the Academy of Sciences [1, 2].

A few words should be said about the history of this work. In 1947, Markov and an American mathematician Emil Post independently established the algorithmic unsolvability of the word problem for finitely presented semigroups. The problem was formulated by A. Thue in 1914, long before the concept of algorithm was formalized. Markov's unsolvable Thue system was given by 13 generators and 33 relations. Tseytin's system had 5 generators and 7 relations:

$$\begin{array}{lcl@{\qquad\qquad}lcl} ac & \Longleftrightarrow & ca & ad & \Longleftrightarrow & da \\ bc & \Longleftrightarrow & cb & bd & \Longleftrightarrow & db \\ ce & \Longleftrightarrow & eca & de & \Longleftrightarrow & edb \end{array}$$

$$cca \Longleftrightarrow ccae$$

To this end, Tseytin used a group with an unsolvable word problem, the existence of which had been established shortly before by a Russian mathematician Pyotr S. Novikov. For more than a decade, Tseytin's example remained the simplest example of an undecidable Thue system. Eventually, Tseytin's ideas were used to construct other simple examples of relevance to computer science.

In 1956, Tseytin received his master degree in mathematics and entered the postgraduate course under Prof. Markov. For some time he worked in constructive mathematics of the Markov–Shanin school. One of Tseytin's significant achievements was proving the non-existence of a constructive real function on an interval that takes values of different signs at its ends but has no constructive root on this interval. On the other hand, he also proved that there is no algorithm that, for any constructive function with the same property, finds a constructive root in the interval.

Gregory Tseytin with his academic advisor Andrei Markov Jr. (1960)

Another remarkable Tseytin discovery was the first nontrivial positive result in this area that consisted in the fact that any constructive mapping of a constructive complete separable metric space into another constructive metric space is continuous (in constructive mathematics, continuity implies the existence of an algorithm that finds δ from ε). This remarkable theorem was a fundamental strengthening of an earlier result by Prof. Markov on the absence of constructive discontinuities of constructive functions.

In 1959, Tseytin started as a research associate at the Institute for Mathematics and Mechanics at the LSU. In 1960, he defended his Ph. D. thesis on algorithmic operators in constructive complete separable metric spaces. His official opponents (examiners) were Vladimir Uspensky and Nikolai Shanin. For several years he continued to study constructive mathematics. Based on new results, partially obtained with his friend Igor Zaslavsky, in 1968 Tseytin defended his second (higher) doctoral thesis [4]. His official opponents were Markov, Boris Trakhtenbrot, and Shanin. In this work, Tseytin summed up his research in the field of constructive mathematics, and never returned to this subject.

Another of Tseytin's major research topics was computational complexity theory. He was a pioneer in the analysis of lower bounds for the time complexity of algorithms and lengths of proofs for propositional formulas. The Markov school worked with Markov normal algorithms, more versatile than Turing machines. (To account for the greater power of Markov algorithms, in 1954, Tseytin enriched the Turing machine appropriately [5].) In 1957, he proved the lower bound $n^2/\log^2 n$ for the time complexity of inverting a word by Markov algorithms. In 1969, Tseytin obtained a tight lower bound $c \cdot n^2$ where the constant c depends on the given inverting algorithm [6].

There are three concepts introduced by Tseytin that are constantly used in the theory of proof complexity.

1. *Tseytin's extension rule* that allows one to introduce new variables and use them to denote arbitrary formulas [3]. This rule makes even a rather weak system, the resolution method, so strong that we still cannot prove exponential lower bounds for the resulting system.

2. *Tseytin tautologies*, systems of linear equations over a finite field constructed according to a given graph. These were the first formulas for which a superpolynomial lower bound for the complexity of propositional proofs was proved. This proof, due to Tseytin, dealt with regular resolutions. This work [3, 9] had a great influence on further studies of the complexity of propositional proofs and became one of the most cited works of the Russian school in mathematical logic and algorithm theory.

3. *Tseytin's efficient translation of propositional formulas into conjunctive normal form*, which became a standard technique in computational complexity theory.

After the completion of his second doctoral thesis, Tseytin's attention switched to computer science. He quickly became the informal leader of the Leningrad programming school. It was most fortunate that a brilliant mathematician became a pioneer and a leader in computer science supporting its autonomy and distinct character. Tseytin pointed out the crucial role of the social dimension in computer science [10]: the collective nature of software creation, the need to adapt the product to human perception, the problem of protecting programs, etc. At that time Tseytin admitted to his colleagues that he no longer considered himself a mathematician.

Tseytin taught numerous courses. A notable example was his course on the theory of algorithms and recursive functions, which was taught over five semesters. His teaching was non-standard. He explained the ideas, tricks and algorithms, but the main emphasis was on complex open problems. He encouraged nontrivial questions, new problems, and new ideas, which he valued more than a simple reproduction of the class material. Students had to put a lot of effort in and between the classes, and many commented that they learned more from Tseytin than from other professors who taught the same course. Here is what some of Tseytin's colleagues had to say:

- Whatever Tseytin did, be it programming or kerosene stove repair, he did it better than anybody else.
- Tseytin's talks for the department were not just interesting, they were always brilliant, a celebration of thought.

Inspired by Noam Chomsky's research on formal grammars, Tseytin became interested in machine processing of texts in natural languages. One facet of this problem was the automatic translation from one language to another. Additional problems emerged that required a deep understanding of the mechanisms underlying natural languages and their description in a form suitable for computer applications. In the 1960s, Tseytin headed a new LSU research group working on experimental systems in this area. Later his group became the Laboratory of Mathematical Linguistics and eventually the Laboratory of Intelligent Systems. At the time, many tried to automate translation from any language to any other. To Tseytin, an avid polyglot and an Esperanto enthusiast, this idea was especially close. Over years, he developed a programming framework based on semantic networks that combined methods of linguistics, formal logic, and systems programming. Automatic translation built on semantic networks has never competed with modern translation tools based on statistical methods. However, Tseytin's methods were capable of extracting the text's meaning and rendering it in applicable areas, such as understanding of natural language, synthesis of computer programs, and artificial intelligence. In the 1980s–90s, using the computers available at the time, the computer scientists working with Tseytin got practical results that were quite advanced even by today's standards.

It was typical of Tseytin to reevaluate his research priorities from the point of view of new results and ideas. In the late 1960s, inspired by the new trends in the theory of algorithmic languages, he focused his research in this field and away from mathematical logic. His 1981 report of this transition titled "From logicism to proceduralism" [8] contains insightful observations about the problems of language processing and artificial intelligence that are still relevant today.

Also at the same time and influenced by the same ideas, a strong group of computer scientists was formed at the Department of Computer Science and the Institute of Mathematics and Mechanics. This group was engaged in the development of compilers for various programming languages. Tseytin was especially interested in Algol-68, a language with a very rich syntax, whose implementation presented a serious scientific challenge. Suffice it to say that only two complete compilers of this language have ever been implemented. Tseytin headed the project at the initial stage and developed the basic ideas; in 1968 the group joined the international cooperation on the development and implementation of this language [7]. Svyatoslav Lavrov who headed the LSU Dept of Computer Science between 1972 and 1988, an eminent authority in mathematical logic and computer science, wrote that it was under the influence of this remarkable work that the Leningrad school of programming was born and developed. He also noted that even if Algol-68 had not found any application, the emergence of the Tseytin programming school alone would have been an excellent justification of the group existence.

The times were changing, and Tseytin began to cooperate with foreign research and development groups engaged in practical computer science. In 1990–1995, Tseytin worked on making improvements to the MS DOS operating system and creating a text database. Next, in 1995–1996, he worked with the Italian company Microstar Software Ltd.

In 1997, Tseytin spent several months at the IBM Almaden Research Center in California doing research in phenomenal data mining. In the spring of 1999, he continued this work at Trinity College Dublin [11] where he also taught data mining.

Later, in 1999, Tseytin participated in a natural language processing project at the University of New Mexico. This project only lasted one year, after which Tseytin decided to stay in the USA for good. With the support of prominent experts from several countries, he applied for permanent residency under the Extraordinary Ability category. One of those experts was the famous Stanford professor John McCarthy who highly appreciated Tseytin's research, promoted Tseytin's 1997 visit to the Almaden Center, and helped Tseytin during his first years in the USA. Tseytin became a permanent US resident in 2002 and an American citizen in 2008.

In 2000, Tseytin moved to the San Jose, CA, area. He first worked at Rational Software Corporation on their Purify Software, a tool for runtime analysis of complex computer software. In 2003, Rational Software was acquired by IBM, where Tseytin worked with the same group until 2009, receiving 4 patents in the process. After that Tseytin worked for 4 years at Stanford University in the project of the philosopher and logician Patrick Suppes on the construction of an automatic learning system for schoolchildren. Eventually, the project matured, was sold to a commercial company, and now is used as part of Redbird Personalized Learning System.

Tseytin was well into retirement age, but he did not wish to retire and continued to interview for other positions. He did some work with a startup company, was a visiting professor at Stanford, volunteered for political campaigns and the advocacy group Indivisible, and was employed by the Census Bureau for the 2020 population count. Unfortunately he never found in the USA an adequate application for his extraordinary capabilities.

Throughout his life, Tseytin was very active in the scientific community. He was an active member of the Leningrad (later St. Petersburg) Mathematical Society almost from the time of its resumption in 1959. Starting in 1989, he became deeply involved in the work of the St. Petersburg Union of Scientists (SPbSU) serving as a member of its coordinating council. After his departure, the SPbSU commissioned Tseytin to be the union's representative in the USA.

Tseytin had a huge and rare gift. People around him admired his mathematical talent, which was realized in many of his remarkable works. His achievements have been universally recognized by the scientific community. Yet, any sense of superiority was completely foreign to him. People were often surprised with his genuine kindness and sincere willingness to help colleagues in any work. His talent was not only in the generation of new ideas, but also in his perseverance in completing their implementation. It is challenging to match rare abilities with suitable professional opportunities, and unfortunately, there are times when some talents are not adequately appreciated. Tseytin had periods when his abilities were not in demand, or economic conditions impeded his professional progress. We will remember this amazing scientist and a wonderful and generous person who, despite formidable obstacles, managed to bring to life so many of his ideas. See Tseytin's main papers at `http://mathsoc.spb.ru/pers/tseytin/bib.html`

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