

KNOW THE PERSON BEHIND THE PAPERS

Today: Peter Winkler

Bio: *Peter Winkler is the William Morrill Professor of Mathematics and Computer Science at Dartmouth College. His research is primarily in combinatorics, probability, and the theory of computing, with forays into statistical physics. He is a winner of the Mathematical Association of America's Lester R. Ford and David P. Robbins prizes and a Fellow of the American Association for the Advancement of Science. In 2019–2020, he served as Distinguished Visiting Professor for the Public Dissemination of Mathematics at the National Museum of Mathematics in New York City. He has written three books on mathematical puzzles, a book on cryptology in the game of bridge, and a portfolio of compositions for ragtime piano.*



Figure 1: Peter Winkler in his office. The quilt behind him was made by his wife, Lois Winkler, from a pattern Peter devised using Charles Radin's recursive pinwheel tiling.

Stefan Schmid: Peter, before we talk about research, there is already some mathematics in the photograph you sent us. What is the story behind the quilt?

Peter Winkler: The quilt was made by my wife, Lois Winkler. The pattern I suggested is based on Charles Radin's recursive "pinwheel tiling." Each tile is a right triangle whose legs have lengths 1 and 2.

What I like about the pinwheel tiling is that it has some remarkable geometric properties. In the full planar tiling, tile edges occur at a dense set of angles. And any two points that are very far apart can be connected along tile edges with total length asymptotic to their Euclidean distance. So it seemed like a particularly appropriate pattern for a quilt in a mathematician's office.

Stefan Schmid: You started out as a mathematician, with a PhD in mathematical logic. How did you find your way into the theoretical computer science community?

Peter Winkler: Almost by accident — and through one of my students! I did my PhD at Yale in mathematics, with a thesis in mathematical logic. I was always interested in questions about what can and cannot be computed, but I wasn't moving in theoretical computer science circles.

After Yale and a period at Stanford, I joined Emory University. There I had a student named Sam Buss. Sam went to a conference and came back with this thick paperback proceedings volume. The conference turned out to be FOCS. Looking through it was a revelation for me.

It wasn't only that I had discovered a whole community interested in many of the things I liked. I also liked the style of the conference. In mathematics, speakers are often invited by name and then talk for a relatively long time about whatever they are working on. In theory of computing, things moved much faster: people competed to present new ideas, talks were short and succinct, and there was an enormous amount of fresh material.

I started going to FOCS, STOC, SODA, and so forth, and got to know the community. I still like that culture very much.

Stefan Schmid: Many readers may know you at least as well for your puzzle books as for your research papers. How did the puzzle books happen?

Peter Winkler: That also happened rather unexpectedly. I had always enjoyed puzzles. Starting perhaps in high school, I carried two little notebooks around in my pocket. One was blue, and in it I wrote a few words to remind myself of good puzzles. The other was brown, and in that one I wrote down jokes — but only the punchlines, because I assumed I would be able to reconstruct the whole joke from the punchline.

So in a sense even the joke book was a puzzle book!

Years later, Klaus Peters approached me at a conference. He had left Springer and founded A K Peters with his wife, Alice. He said something like, “I hear you have a great collection of mathematical puzzles. Why don’t you email them to me and I’ll publish them?” He made it sound very easy. And, in fact, he made the process about as easy as publishing a book can be.

Once the first book appeared, people began emailing me puzzles from all over the world. That became a wonderful source of material for the later books. I always try to credit the person from whom I heard a puzzle, whether they invented it themselves or passed it along.

Stefan Schmid: What makes a puzzle a *good* puzzle for you? Is it mainly about being difficult?

Peter Winkler: No — that’s exactly the misconception I would like to fight.

People often think puzzles are challenges or tests: What’s the hardest puzzle? Can you solve it? They can even become nervous when somebody tells them a puzzle because they fear they are going to be tested.

That is not how I think about puzzles. A good puzzle asks a question that you might never have thought of asking and gives an answer that you might never have expected. Most importantly, it should be entertaining. It should be neat. It should communicate an idea.

In fact, sometimes the puzzle works even better if you *don’t* find the answer yourself. Puzzles for examinations or competitions have a different purpose; they are there to test you, and that’s perfectly legitimate. But the puzzles I collect for my books are not there to test people. They are there to amuse them, get them excited, and show them something they didn’t know before.

Stefan Schmid: Can you give us an example of such a puzzle?

Peter Winkler: Here’s one that reached me indirectly from Japan. The question is wonderfully short:

Can any ten points in the plane be covered by pairwise disjoint unit disks?

Every word matters. If the disks did not have to be disjoint, the problem would be trivial. If you had a huge number of points packed into a fine grid, you clearly could not cover all of them by disjoint unit disks. But for a small number of points it seems plausible.

The exact largest number of points that can always be covered is not known. But there is a beautiful probabilistic argument showing that ten points can always be covered.

Take the densest familiar packing of unit disks, the hexagonal or honeycomb packing. Its density is

$$\frac{\pi}{\sqrt{12}} > 0.9.$$

Now shift the entire packing uniformly at random. Any fixed point is covered with probability $\pi/\sqrt{12}$. So for ten fixed points, the expected number covered is more than nine.

Because the number covered is an integer, some shift must cover all ten.

That's the kind of proof I love. It's a question I would never have thought of asking, followed by a very short argument that makes you see something differently. Paul Erdős would have liked it.

Stefan Schmid: Do puzzles ever turn into research papers?

Peter Winkler: Absolutely. A very recent example began with a high-school student, Tejo Madhavarapu. I was on Zoom with Tejo and Kyle Petersen from DePaul University, and Kyle mentioned that Tejo composed puzzles. I asked him to send me some favorites.

He sent only five. Two of them were puzzles I had never seen and really liked. That's an extraordinary success rate — when people send me puzzles, perhaps one in a hundred really excites me.

One of Tejo's puzzles was called "Is There an Ideal Color Wheel?" Think of the familiar six-color wheel from elementary school, where, for example, blue and yellow mix to make the green between them. Tejo asked whether one could arrange six colors so that *every* color is obtainable by blending its two neighbors, without all six colors being identical.

What is striking is that the problem can be abstracted away from color almost completely. It connects to the familiar fact that a harmonic function on the vertices of a finite connected graph must be constant. We found that one can formulate and prove a much more general statement on finite graphs without needing the usual machinery of averaging. That puzzle became a joint research paper with Tejo and Kyle.

I like that transition very much: a question posed for fun reveals a mathematical structure, and suddenly you're doing research.

Stefan Schmid: You also composed a probability puzzle about baseball that seems designed to trick one's intuition.

Peter Winkler: Yes. Imagine two equally matched baseball teams, except that each has the same small home-field advantage. They play a best-of-seven series: the first team to win four games wins the series. Suppose Team A hosts the first three games and Team B hosts the remaining four.

The obvious argument says that the series lasts fewer than six games on average, so more games are actually played at A's home. Therefore A should have the advantage.

I used this puzzle in a course called *Probability and Intuition* that I taught while I was Distinguished Visiting Professor at the National Museum of Mathematics in New York. We met over dinner with a group of people from many professions — finance, medicine, law, journalism, film — and discussed probability through puzzles.

Everybody came back with essentially that argument: Team A should have the advantage.

Then one of the participants, a cardiologist, raised her hand and said: Suppose they simply play all seven games, even after somebody has already won four. The extra games cannot affect who wins the series. But if all seven games are played, Team B obviously has the advantage because four games are at B's home and only three at A's.

And that's correct: Team B has the advantage.

The nice paradox is that if the teams play the series every year for a very long time, Team A will win more individual games, while Team B will win more series.

Someone asked how home-field advantage in game seven, which is rarely needed, can be as valuable as home-field advantage in game one, which is always played. My analogy was transportation in Manhattan: having a car available exactly when you need one can be just as useful as owning one all the time. Game seven is there when you need it.

Stefan Schmid: Where and when do you tend to get your best ideas?

Peter Winkler: In the morning, in the shower.

If I'm going to have a good idea during the day, there's a good chance it will happen during those five minutes or so, when my brain is waking up. I realize there is a perfectly good argument for showering in the evening: after a long day, you get clean and then sleep in clean sheets. But if I switched to evening showers, I fear I would have missed a lot of good ideas!

More seriously, ideas are often triggered by something accidental. The important thing is what Erdős used to call keeping your "brain open."

He liked to tell a story about the discovery of X-rays. According to Erdős, Crookes and his students had already noticed that photographic plates behaved strangely near a Crookes tube. Crookes's reaction was essentially to tell his students not to leave photographic plates near the tube. Röntgen, faced with the anomaly, asked what was going on.

I can't vouch for the historical accuracy of the story, but the lesson is excellent: when something unexpected happens, don't automatically treat it as an annoyance. Your brain should be open enough to notice that it might be telling you something.

Stefan Schmid: That sounds like a philosophy for choosing research problems as well. How do you decide what to work on?

Peter Winkler: I'm not sure anyone should take my advice on this, because I have shown a certain absence of self-discipline in choosing research problems.

I get interested in a question. If it then leads into an area that is not one of my mathematical strengths, the disciplined thing would be to give it up and choose a problem that better matches my expertise.

I don't.

That's one reason I started in mathematical logic, then moved into combinatorics, probability, theory of computing, and even some statistical physics. It seems to suit my personality, and somehow I've gotten away with it.

For students, though, my advice is somewhat different: try to get really good at *something* first. Having a strong base gives you more freedom later to wander.

Stefan Schmid: And what do you do when you get stuck?

Peter Winkler: Most of the time you're stuck. That's the steady state for a researcher.

If you really hit a wall and don't see what to do next, put the problem aside and work on another one. Or watch a soccer game. That was actually my plan for later on the day we spoke.

Sometimes the problem you're having can even suggest another research problem. At Bell Labs, I had a wonderful intern, Ioana Dumitriu. We were trying to decide what project to work on, and that led us to ask a question about random walks: suppose several tokens are walking on a graph toward a target, and at every step you are allowed to choose which token makes the next random move. Which token should you move?

If you had to choose one token forever, you would simply choose the one with the smallest expected hitting time. But if you can change your mind after seeing where a token moves, the problem looks much more complicated.

It turned out that there is still a remarkably simple index policy: assign a certain value, which we called a "grade," to each state and always move a token with the best grade. We later realized that this was related to the Gittins index. The resulting paper was called *On Playing Golf with Two Balls*: hit one ball, and if you don't like where it goes, hit the other one.

The funny part is that we forgot to explain the golf analogy anywhere in the paper. Golf and balls occur only in the title.

Stefan Schmid: How do collaborations happen for you? Do you deliberately seek out particular collaborators?

Peter Winkler: Not very deliberately. A lot of it is happenstance. You talk to somebody, you both get interested in the same problem, and something happens.

I've written many papers with Graham Brightwell. We've hardly ever been at the same institution, but somehow the combination works. There's a certain magic to a good collaboration that I don't know how to explain. The same has been true of my work with László Lovász.

On the other hand, there are people I admire enormously whom I've known for decades and with whom I've barely worked. I've known Noga Alon since he was a student. I remember him showing me a beautiful combinatorial proof using exterior algebra. I told him I knew essentially nothing about exterior algebra beyond the definition. He said, "That's all you need," and showed me the proof.

We eventually ended up on a joint paper, but even there we had worked on different ends of the problem and were connected through the other coauthors. Collaboration is often less planned than people imagine.

Stefan Schmid: You mentioned teaching. Do puzzles find their way into your courses?

Peter Winkler: I can't help it.

I tell myself I'm only going to introduce a puzzle when it's exactly relevant to what I'm teaching. In practice, of course, puzzles have a way of appearing.

I teach a course called *Topics in Probability* about once a year, and I can essentially choose the subject. One year it might be percolation, another random graphs and graphons, another game theory or decision theory. Recently I taught permutations and sorting, partly because I've been interested in permutons — limit objects for permutations, analogous to graphons for graphs.

That course gave me an excuse to use one of my favorite probability examples, Pólya's urn. Start with one coin in each of two urns. Add coins one at a time, with the probability of choosing an urn proportional to the number of coins already in it. Now imagine doing this until there are a million coins, and ask students how much they would pay in advance for the contents of the *smaller* urn, at one dollar per coin.

People are extremely cautious. In one class, collectively, they were willing to invest about ten dollars.

But the expected value of the smaller urn is about a quarter of a million dollars.

There is a lovely proof using a shuffled deck of cards. Insert cards one by one into uniformly chosen positions. Track only how many cards lie above and below the first card. The evolution is exactly the urn process. Since the first card ends in a uniformly random position, the final split between the two urns is uniform. Again, the point is not to test the students. It's to give them an intuition they won't forget.

Stefan Schmid: Students now have another tool available: generative AI. Have you experimented with it for mathematics?

Peter Winkler: Yes, and I find it fun and occasionally useful.

It is especially helpful when I ask it to compute something that I can then verify independently. If I ask an AI system to prove something, I don't trust the answer — I have to prove it myself anyway.

I've also had amusing conversations with the systems. Claude tends to be very apologetic when I point out an error and eventually starts asking me for hints. Then I have to say, "This isn't a puzzle. I don't know the answer either. I'm telling you everything I know!"

I'm impressed by some of the recent work where stronger systems have contributed to mathematical problems. But at the moment I mostly regard AI as another interesting tool, one whose output needs to be checked carefully.

Stefan Schmid: You spent a year in Germany as a Humboldt Fellow. What do you remember from that time?

Peter Winkler: I spent 1984–85 at TH Darmstadt, which had a very good combinatorics group. The Humboldt Foundation was a wonderful host. They didn't just bring you to Germany and support your research; they organized trips and social events where Humboldt fellows from around the country could meet.

I also remember being surprised by the Christmas payment. I received what seemed like two months' salary around Christmas, which was completely unheard of to me as an American.

With a name like Winkler — and my middle name is Mann — people naturally expected me to speak German. My family with the Winkler name had come to the United States in the nineteenth century from Bessarabia, in what is now Moldova, so the German language had long since disappeared from the family.

I did learn enough German in Darmstadt that I once gave a talk in German. I can truthfully say that I did it. I can also truthfully say that it was a bad talk in German!

Stefan Schmid: Do you have a message for the theoretical computer science community? Anything we should be doing differently?

Peter Winkler: I'm delighted with the theory community. I think it's a great community, so I'm not going to give it advice — it seems to be doing very well without my help.

Here at Dartmouth we have a Theory Reading Group with colleagues and students from mathematics and computer science. That's something I enjoy very much: the community is broad enough that probability, combinatorics, algorithms, security, and other areas naturally interact.

So I'm happy to leave the community alone and let it keep doing what it's doing.

Stefan Schmid: Before we finish, you said there was one more STOC story you wanted to tell us.

Peter Winkler: Yes. Graham Brightwell and I proved that counting linear extensions of a partial order is $\#P$ -complete. This is an interesting counting problem because every finite partial order has a linear extension — the difficulty is not deciding whether one exists, but counting how many there are.

When I arrived at STOC in New Orleans, someone remarked that it was surprising that our reduction went all the way back to $\#3$ -SAT instead of starting from a problem that looked more similar.

That reminded me of a story about a theory course at Berkeley. A student was asked on the final exam to prove that Hamiltonian Path is NP-complete. He spent the whole exam trying to reduce directly from 3-SAT, even though he could have obtained the result almost immediately from Hamiltonian Cycle.

The professor — Umesh Vazirani, as I remembered after our interview — asked him why on earth he had done that. The student replied that his undergraduate advisor had been Christos Papadimitriou, and Christos had told him:

“Real men reduce from SAT.”

So for my STOC talk I went to a shop near the hotel and had a T-shirt made saying exactly that.

It got a small laugh. Then I explained that I really should have put “— Christos Papadimitriou” underneath it, but the shop charged 25 cents per letter and I would have had to sell my car.

That got a much bigger laugh.

Stefan Schmid: Peter, many thanks for this wonderful conversation. It seems appropriate that an interview about puzzles ends with a complexity-theory joke.

Peter Winkler: It was fun to talk to you. I hope you find something useful in all of this!