

VIEWPOINT COLUMN

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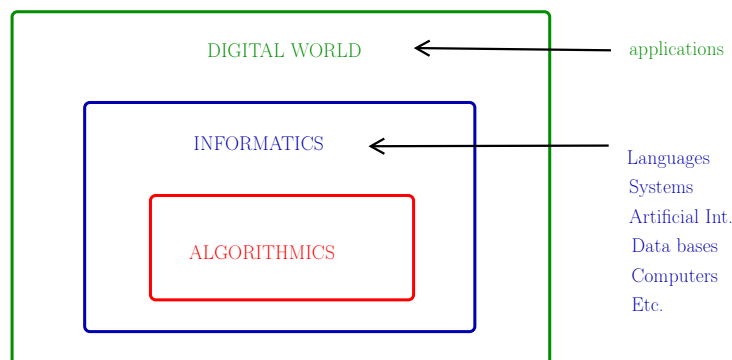
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Informatics Is a Very Old Science!

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What is an algorithm? *Informatics* is the science of algorithms and their applications¹. If we suppress algorithms, informatics disappears! Informally an algorithm is a *finite sequence of well-defined basic operations* that, under predefined assumptions (defining the context in which the algorithm has to be executed) describes what has to be done (by a human being or a machine) to obtain a result. It is defined by a specification which describes precisely what we want to obtain (e.g. bisect an angle, or sort a set of words in alphabetical order). So an algorithm manipulates symbols (not only numbers or letters), it has inputs and produces outputs. When the inputs and outputs are data, an algorithm can be seen as a data transformer producing an information which has a meaning defined by the specification of the algorithm.²



As a very simple example of an algorithm let us consider the multiplication of any pair of integers x and y . We assume that the only basic operations the algorithm can use are restricted to the multiplication of an integer by 2, the division of an integer by 2, and the addition of any pair of integers. To simplify the reading of the algorithm we consider there is a two-column array in which integers can be written. The algorithm is the following sequence of basic computation steps:

¹According to Leibniz a concept or a specific domain must be identified by a single name (word) and not by a set of words (which can be its definition). That is why I use the word *Informatics* defined as being the *science of computing*. This is consistent with the words we are already using such as medicine, chemistry, biology, mathematics, geology, archeology, philosophy, literature, etc.

²In that sense sewing [7] and origami [17] are algorithms. A sequence of weaving cards precisely defines an algorithm (that can be executed by looms). Differently cooking recipes are not algorithms. This is because (among many other issues) the same recipe used by different cooks produces different results!

- We first write x and y at the top of the left and right columns respectively.
- Then, in the left column we write the integer part of the previous value divided by 2, and this is repeated until we obtain the value 1.
- Then, in the right column we write the previous value multiply by 2, and this is repeated until the column is filled.
- Then we cross out in the second column all the numbers associated with an even number on the left column.
- Finally we add all the numbers of the second column which have not been crossed out. This number is the result.

This multiplication algorithm is called *Russian multiplication*. It is used in some computers where, in binary notation, multiply (divide) by 2 consists in a left (right) shift. As we can see, this multiplication algorithm does not use the Pythagorean table traditionally used in many countries³. An example is depicted below namely the computation of 41×272 . The first table corresponds the division and multiplication by 2 as previously described. The second table describes the crossing out of values. The addition of the remaining non-crossed out values provides the result, namely $272 + 2176 + 8704 = 11\ 152$.

41	272
20	544
10	1088
5	2176
2	4352
1	8704

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$$41 \times 272 = 272 + 2176 + 8704 = 11\ 152.$$

It is worth noticing that this multiplication algorithm, which relies on base 2, was known and used in Egypt by the scribes and priests in the time of Pharaohs. With our modern notations, the proof of this algorithm is pretty easy. We give it for the specific case of 41×272 , but it can be easily generalized to any pair of integers. Using decomposition of integers in base 2 we have (remember $1 = 2^0$):

- $41 = 2^5 + 2^3 + 2^0$,
- $272 = 2^8 + 2^4$,
- $(2^5 + 2^3 + 2^0) \times (2^8 + 2^4) = 2^{13} + 2^{11} + 2^9 + 2^8 + 2^7 + 2^4$,
- and finally we obtain $2^{13} + 2^{11} + 2^9 + 2^8 + 2^7 + 2^4 = 11\ 152$.

³The Pythagorean table gives for free the product of any pair of integers in the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. There are many algorithms to multiply any two numbers.

On the powers of 2: Mystery or something deeper?

The sequence of consecutive integers from 10 to 14 has the following surprising property

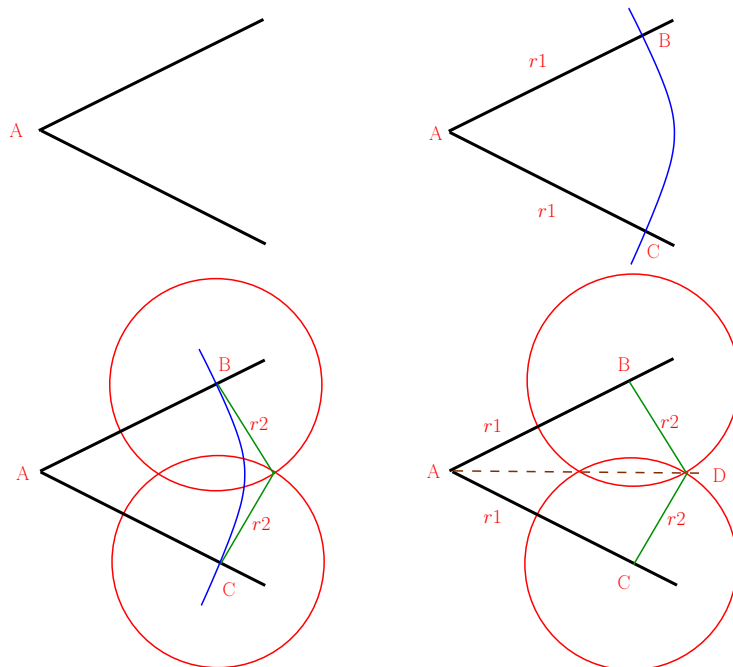
$$10^2 + 11^2 + 12^2 = 13^2 + 14^2 = 365$$

which is the number of days in a calendar year! Sorcery or amusing anecdote?

Theorems (Mathematics) vs Algorithms (Informatics) There is a strong analogy between theorems and algorithms, namely each is made up of two parts: a statement and an associated correctness proof. Moreover in both cases –theorem/algorithm– it is possible to have different proofs for the same statement. A point in which they differ lies in the fact that, due to its operational nature, an algorithm assumes an underlying notion of time (needed to execute it), while a theorem does not.

In mathematics books, algorithms devoted to geometry are called constructions. As an example let us consider the bisection of an angle with the operations provided by a ruler (draw straight line segments) and a compass (to measure segment lengths and draw circles). This is one of the geometry problems stated and solved by Ancient Greeks. It consists in dividing an angle in two equal parts, and this has to be done with a ruler and a compass (those constitute the only basic operations that can be executed). Let us consider the angle (left top of the figure). The algorithm is the following sequence of operations:

1. Draw an arc of circle of center A of any radius, say r_1 . This arc cut the angle in B and C.
2. Let us now draw two circles whose centers are B and C such that they intersect (let us note that this is always possible). Let r_2 this radius.
3. Claim the line AD bisects the angle $\angle BAC$.



Proof of the claim. We have $|AB| = |AC| = r_1$ and $|BD| = |CD| = r_2$, from which it follows that the triangles ABD and ACD are equal, and consequently the corresponding angles $\angle BAD$ and $\angle CAD$ are equal. QED.⁴

⁴The trisection of an angle with only a ruler and a compass is impossible. The notation π , due to William Jones

On the use of integers and letters as symbols: Any representation needs an interpretation

Inherited from the Indians the ten digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) allow us to write any number. Differently, the symbols used by the Roman were a set of seven letters, namely I (1), V (5), X (10), L (50), C (100), D (500), M (1000). Then the composition of these letters, satisfying some constraints, denotes an integer. As examples, 1914 is written MCMXIV, 42 is written XLII, 914 is written MCMXIV, 42 is written XLII, 264 is written CCLXIV. It is important to see not all the combination of letters produces an integer. As an example XXVIC is meaningless. It follows from this coding system that multiply XLII (42) by CCLXXII (272) is not an easy task. Doing arithmetic with such a notation is nearly impossible.⁵ Simple notations and symbols are crucial to make life easier.

The meaning of a notation depends on the context. Given a street, the assignment of increasing integers to a sequence of houses makes easy to find the location of a given house. But house numbers are symbols that cannot be added or multiplied. They are not mathematical integers (the multiplication of two house numbers does not give a house number).

A phone number is the name of a phone, but this name has a different meaning according to the type of phone. A landline phone addresses a physical location such as a house or an office. Differently a cell phone is associated with a single person (and not a location).

It is worth noticing that (according to [5]) 30,000 years ago, the symbol used by human beings to represent small integers (1 to 5) were sequences of painted fingers as evidenced by the drawings of the Cosquer and Gargas famous caves, i.e. long before the invention of writing.

The geographic origin of algorithms According to today knowledge it seems that the very first algorithms were designed in Mesopotamia, Egypt, China, India and Peru. Their aims were to solve problems related to inheritance, law (see Hammurabi code), field area computation, economy (exchanges money/goods), and interest rates. The interested reader will consult [8, 10, 20].

The origin of the word algorithm The word *algorithm* comes from Al Khwarizmî (which means that the origin of his family was the region of Khwarem in Uzbekistan). Born in Khiva around 780, he died in Baghdad around 850. He lived during the golden period of the *House of Wisdom* (at that time the Calif of Baghdad was Hârûn ar-Rachîd ben Muhammad ben al-Mansûr). It is worth noticing that the book *The Thousand and One Nights* was written in the very same period.

Al Khwarizmî was a polymath interested in many scientific and non-scientific topics: arithmetic, geometry, astronomy, geography, history, Jude calendar. He is the person who introduced Indians numbers in Baghdad and Arabic countries. He was also interested in astrolabes and created a “start-up” to build and sell astrolabes. As a mathematician and astronomer he computed a lot of astronomical tables. He wrote books on many other domains. His main book

in 1706, was made popular Euler. The transcendence of π was proved by Ferdinand von Lindemann in 1882 by showing that it is impossible to build a square that has the same area as a circle of a given radius using only a ruler and a compass.

⁵It is interesting to notice that the number obtained by writing all the symbols (but M) in their decreasing order, namely DCLXVI, is the number 666 attributed to the *Beast*: hazard!

was the *Al Kitab Al-Jabr wa'al-Muqabala*. This is the book that gave rise to the word *algebra*. It is in this book that are presented algorithms to solve what we call today quadratic equations.

To attain this goal he identified six patterns of quadratic equations produced six algorithms, one for each pattern. Such a pattern was a construction of a square from rectangles and triangles obtained from the explicit numbers appearing in the equation. Then he tried to solve cubic equations, without success. This was successfully done by Omar Khayyam (1048-1131), who was also the great poet who wrote the *Rubaiyat*.⁶

Mathematics vs physics The goal of physics is to understand, model and explain the phenomena of the universe. For example, the formulas $F = G \frac{m \times m'}{d^2}$ and $E = mc^2$ are not mathematics but physics. The first captures the law of universal gravitation between two bodies; m and m' are the masses of the two bodies, G is the gravitational constant, and d is the distance between the two bodies. The second captures the equivalence between the mass m of a body and the energy E associated with it (c is the speed of light). In both cases, these are physics formulas and not mathematical formulas, namely they are a conceptual tool that has made it possible to arrive at these physics formulas that enrich our knowledge of the world in which we live.

In the same way, since Turing, thanks to their power, the algorithms have become inescapable and are no longer restricted to their uses in mathematics. As already said, they constitute the heart of *Informatics*, which, in addition to algorithms, includes all its applications).

When a new scientific domain acquires a name A new scientific domains first appear informally and then become autonomous when great results are produced. More precisely, each scientific domain has its

geniuses who produced fundamental results that provide us with deep foundations such that, for each domain, there is the period before and the period after. A few examples appear in the table below.

Physics	Newton (1642-1727)
Chemistry	Lavoisier (1643-1794)
Informatics	Turing (1912-1954)

Mathematicians vs informaticians While one of the aims of science was to “put the world into equations”, we know today that everything cannot be captured by a formula. So, an aim of today science is to put the world into equations and algorithms. Said in another way, Galileo Galileo said “the great book of nature is written in mathematical language”. This can be true for physics, not for other sciences such as life sciences. Today, the pair made up of mathematics + informatics seems to be the language of all sciences. As said by Claude Berrou (co-inventor of Turbo-codes) *Informatics is to science what language is to intelligence*.

⁶While Al Khwarizmi had the intuition that solving quadratic equations was related to a two-dimensional space puzzle (to solve with squares, rectangles and triangles), he did not have the intuition that solving cubic equations could be related to a tri-dimensional space.

Among others, informatics and mathematics present an important difference, namely mathematics does not have the touch “run” which transforms an algorithm (written in some programming language) into a finite execution! This has a fundamental consequence, namely informatics evolves in the field of finite time and finite memory. As an example, while mathematics (correctly!) considers π (the Archimedes number) as having an infinite number of digits, any algorithm using π is restricted to consider a finite approximation of it. ⁷

To illustrate the previous claim let us consider the following two objects: the object denoted “1+2” and the object denoted “3” with their usual meaning, and let us ask the question: “are these two objects the same object or are they different objects?” Nearly all mathematicians answer “it is the same object”, while nearly all informaticians answer “they are different objects”. This is due to the simple observation that informaticians see “1+2” as an algorithm which needs to be executed to obtain a result, namely “3”. These answers show that answering a question has a bias depending on the cultural and scientific contexts of the person who has to answer. Algorithms are the roots which gave rise to informatics, and this has a long history that started a very long time ago.

ps1. The references not cited in the text are for the readers interested in more developments on the nature and the history of algorithms. The readers interested in the history of *computers* (machines that can execute algorithms) is invited to read the very interesting book [9].

ps2. Computing is not restricted to sequential computing. Parallel computing consists in identifying independent data so that they can be processed simultaneously on a multiprocessor. So the aim of parallel computing is efficiency⁸. Distributed computing (DC) is different: it is the science of *formalized cooperation* among a set of distinct computing entities that need to cooperate to achieve a common goal. DC was introduced by E. W. Dijkstra in 1965 with the definition of the *mutual exclusion* problem. Since then, thanks to its theoretical foundations DC has become an important domain of informatics. Its most famous problem is *consensus* in the presence of failures. It is shown in [16] that consensus and mutual exclusion can be seen as two sides of the same coin : one suited to physical objects, the other suited to logical objects (objects that are implemented by sequences of bits).

ps3. Informatics is made up of theory and technology. Theory is what allows us to understand the deep nature what we are doing. Technology is what should make our lives easier.

ps4. From Paul Valéry (1871-1945):

Modern life tends to spare us intellectual effort just as it does for physical effort. For example it replaces imagination with images, reasoning with symbols, writing with mechanics, and often with nothing at all. It offers us every facility and every shortcut to reach our goal without having to travel the whole way. And that is great, but quite dangerous...

⁷As a simple example it is possible to use a finite prefix of the nice Gregory-Leibniz formula which, as it is infinite, is not an algorithm, namely $\pi = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} \dots$

As humorously said in a popular joke, infinity/eternity is very long, mainly at the end!

⁸Any parallel algorithm could be executed on a mono-processor with the help of an appropriate scheduler.

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